Multi-objective Optimization of Inverse Problems using a Vector Cross Entropy Method

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Abstract —To develop an effective vector cross entropy method, multiple probability density functions are introduced. A measure to consider both the number of improvements in the objective functions and the amount of improvements in a specified objective function is proposed. An information sharing mechanism is also designed. To demonstrate the feasibility of the proposed algorithm, the numerical results on a high frequency inverse problem are reported.

I. A Vector Cross Entropy Method

Inverse problem of design optimizations in electrical engineering usually involves several seemingly conflicting and incommensurable objectives. There is generally no unique solution for such problems and the final solution is a set of compromises of different objectives called Pareto-optimal solutions. Usually the goals of multi-objective optimizations are to minimize the proximity of the final solutions to the true Pareto-optimal front while maintaining the diversity among these found solutions. To achieve these goals, different evolutionary algorithms are proposed [1],[2]. However, only lukewarm efforts are devoted to ensure there is good balance between the conflicts regarding convergence towards the Pareto front, and the requirement to maintain diversity in the found Pareto-optimal solutions [3]. In this respect, a Cross Entropy (CE) method is extended to address the aforementioned issues. Due to space limitations, further details about the scalar CE methods are referred in [4],[5] and hence not repeated here.

A. Multiple Probability Density Functions

A single Probability Density Function (*pdf*) is used in existing scalar CE methods to generate the whole population. However, the employment of a single *pdf* will reduce the diversity and global search ability of the algorithm. In multi-objective optimization study, this means the diversity of the found Pareto solutions is compromised. To address this issue, multiple *pdf*s are proposed, i.e., every individual uses a different *pdf* to generate its own children. Moreover, the normal distribution function $N(\mu,\sigma^2)$, with its mean μ and standard deviation σ, is selected as the *pdf*s. To enhance the diversity of the algorithm, each individual uses the latest N solutions being explored to update its *pdf*. To realize such objective, an archive AP(i) is proposed to memory the latest solutions each individual i has explored. More specifically, in the iterative process, μ^k and σ^k of the k^{th} individual are updated using the ρN best elites found by individual k in the latest N generations according to

$$\begin{aligned}(\mu_{ij})^k &= \sum_{i\in \boldsymbol{I}} x_{ij} \Big/ (\rho N)\\ (\sigma^2_{ij})^k &= \sum_{i\in \boldsymbol{I}} (x_{ij}-\mu_{ij})^2 \Big/ (\rho N)\end{aligned} \quad (1)$$

$$\begin{aligned}(\boldsymbol{u}_t)^k &= \alpha_t(\boldsymbol{u}_t)^k + (1-\alpha_t)(\boldsymbol{u}_{t-1})^k\\ (\boldsymbol{\sigma}_t)^k &= \alpha_t(\boldsymbol{\sigma}_t)^k + (1-\alpha_t)(\boldsymbol{\sigma}_{t-1})^k\end{aligned} \quad (2)$$

where, t is an index for the generations, ρ is the rarity (percentage of *elite* solutions) parameter, α_t is the smoothing parameter which is automatically updated in the proposed algorithm to guarantee there is a good balance of the exploitation and exploration searches using

$$\alpha_t = \alpha_{\min} + \frac{\alpha_{\max}-\alpha_{\min}}{t_N} t \quad (3)$$

where $\alpha_{\min}$ and $\alpha_{\max}$ are, respectively, the minimal and maximal values of the smoothing parameter.

B. Considering Dominations and Improvements

To produce a set of Pareto solutions, the ranking approach is commonly used [6]. However, this approach only determines qualitatively the relationship of dominances and cannot measure, quantitatively, the number of improvements in the objective functions. The amount of improvements in a specified objective function cannot be quantified either. To address this issue, a metric to measure the improvement in these two aspects is proposed. Precisely, for a specific individual x_k, the proposed metric is given by

$$\Delta_{improve}(x_k) = \sum_{\substack{j=1\\ j\neq k}}^{N} \sum_{l=1}^{m} sign[f_l(x_j)-f_l(x_k)][f_l(x_j)-f_l(x_k)] \quad (4)$$

where; x_j *and* x_k are members of the solutions explored by the same individual in the latest N generations; m is the number of objective functions; $sign(x)$ is defined as:

$$sign(x) = \begin{cases} 1 & (x>0)\\ 0 & (\text{otherwise})\end{cases} \quad (5)$$

Incorporating this metric to 'penalize' the fitness value of individual x_k yields the following formula

$$f_{fit}(x_k) = w f_{fit}^{nor}(x_k) + (1-w)[\Delta_{improve}(x_k) / \sum_{j=1}^{N} \Delta_{improve}(x_j)] \quad (6)$$

where, $f_{fit}^{nor}(x_k)$ is the normally defined fitness of x_k, w is a weighting constant between [0,1].

C. Information Sharing

To increase the convergence speed of the algorithm, an information sharing mechanism is designed. Indeed, when the quality of the so far found "Pareto solutions" has not been improved for a predefined consecutive number of generations, every individual, say individual i, will update its archive AP(i) to share the searched information with those of its neighbor individuals. It should be pointed out that the "neighbor" is defined in terms of the Euclidean distances. In the information sharing phase, for an individual, its archive is updated to include the most N best solutions among the archived members of itself and those of its neighbors. After information sharing, the algorithm will keep the archives unchanged for some consecutive generations in order to find better Pareto-optimal solutions effectively and efficiently.

II. Case Study

To evaluate the performances of the proposed vector CE method, it is numerically experimented on different inverse problems including high- and low-frequency ones. Due to space limitations, only an experiment on a high frequency inverse problem is reported. In essence, the problem is to synthesize a non-uniformly spaced antenna array, as shown in Fig. 1, to produce a desired field pattern. In this case study, the desired field pattern is defined as [7]:

$$F_{desired}(\cos\theta) = \begin{cases} \text{cosecant}(\cos\theta) & (0.1 \le \cos\theta \le 0.5) \\ 0 & (elsewhere) \end{cases} \quad (7)$$

$$MSLL_{Desired} \le -22 \text{ dB}$$

where *MSLL* is the Maximum SideLobe Level.

To produce a radiation pattern which is as close as possible to the desired one, the first objectives is selected as:

$$\min f_1 = \sqrt{\frac{\sum_{i=1}^{N}[f_{desired}^{norm}(\theta_i) - f_{designed}^{norm}(\theta_i)]^2}{\sum_{i=1}^{N}[f_{desired}^{norm}(\theta_i)]^2}} \quad (8)$$

where, $f_{desired}^{norm}(\theta_i)$ is the value of the normalized desired radiation pattern at the sampling point θ_i, $f_{designed}^{norm}(\theta_i)$ is the value of the normalized radiation pattern produced by a designed array of *M* elements, *N* is the number of total sampling points.

To produce a field pattern with a minimum possible SideLobe level, the second objective of the proposed study is to minimize the *MSLL*. Consequently, the two-objective synthesis problem becomes:

$$\min \begin{cases} f_1 \\ f_2 = MSLLdesired \end{cases} \quad (9)$$

To quantitatively evaluate the performance of a vector optimizer, a *convergence* measure γ is used as the metric to measure the convergence of the found solution set toward a known set of Pareto solutions, and the *displacement* metric is employed to gauge the uniformity or diversity performance of the found solutions over the non-dominated front [8]. For performance comparison, the proposed CE algorithm and a tabu search method [9] are used to study the problem. It is found that for a typical run, the proposed CE method requires 29655 iterations to find 118 Pareto solutions, which is compared to 34568 iterations in finding 114 Pareto solutions for the tabu searched algorithm. The finally found two Pareto sets of the two algorithms are compared in Fig. 2. To intuitively compare the quality of the two Pareto sets, the Pareto front between the middle two circles of Fig.2 are enlarged in the inset. The parameters γ and *displacement* of the final solution for the proposed algorithm are, respectively, 0.000015 and 0.00024; while the metrics γ and *displacement* of the final solution for the tabu search method are, respectively, 0.000022 and 0.00031. From both the *displacement* metric and the enlarged image in the inset of Fig. 2, it can be inferred, by virtue of the improvements realized by the proposed vector CE method, that the diversity of the final solutions of the proposed algorithm is better than that of the tabu search method. Moreover, from the primary numerical experimental results for the reported case study, it can be seen that the proposed algorithm outperforms the performances of tabu search method in terms of both quality and diversity of the finally found Pareto sets.

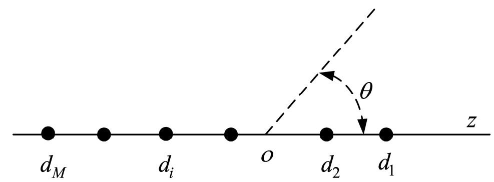

Fig. 1. The configuration of a M-element array placed on the z-axis.

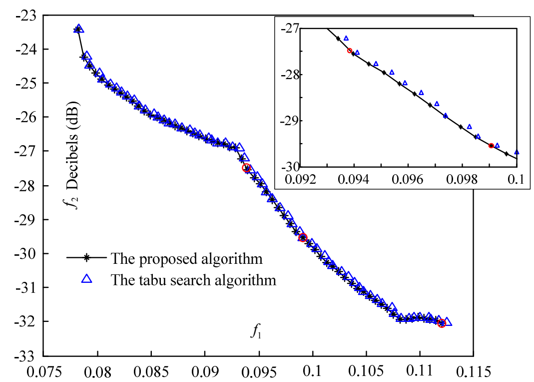

Fig. 2. The final solutions of the proposed and the tabu search algorithm.

III. References

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